

OMAE2013-11015

AN INVESTIGATION OF THE TOLERANCE OF RISER FATIGUE TO CORROSION PITTING

Yetzirah Urthaler
BP America Inc.
Houston, TX, U.S.A.

Mark Cerkovnik
2H Offshore Inc.
Houston, TX, U.S.A.

Fengjie Yin
2H Offshore Inc.
Houston, TX, U.S.A.

David Saldana
2H Offshore Inc.
Houston, TX, U.S.A.

Robin Gordon
Microalloying Inc.
Houston, TX, U.S.A.

ABSTRACT

When risers are designed it is common for corrosion to be accounted for by including a corrosion allowance in the wall thickness [3]. However, when designing risers which are subject to fatigue loading from various sources, simply allowing extra thickness in the wall is inadequate to protect against the accelerated fatigue crack growth driven by corrosion.

This paper illustrates a methodology for assessing the fitness for service of a steel catenary riser with various levels of pitting corrosion. The methodology uses FEA tools, as well as classical fracture mechanics, to predict the rates of crack growth and arrive at predictions of life. Once corrosion begins and pits form, the structure may experience an increase in crack growth rate, caused by the influence of the chemistry of the produced fluid on the steel and by the stress effects of the pit geometry. Further complications arise if extreme storms cause riser stresses to exceed yield, which then requires the use of strain based methodology.

The results of the illustrative study demonstrate that riser designs should account for the potential of accelerated crack growth where there is a potential for pitting corrosion, and that by only adding a corrosion allowance to account for loss of burst capacity, an inadequate design can easily result.

Keywords: Corrosion, Pitting, Fitness for Service, SCR, Riser, Fatigue

INTRODUCTION

Subsea riser design is one of the most challenging parts of any deepwater field development. Risers are pipes connecting a floating platform with the seabed and are used as conduits for transporting crude. The dynamic operational and environmental loads experienced by the risers make risers fatigue critical

structures. With internal contents that are often corrosive and seawater external, the fatigue risks can be compounded if corrosion control is lost; even temporarily.

Marine production risers represent a special design challenge for fitness for service evaluations because they may see high stress, fatigue loading, as well as a corrosive environment. Some risers are designed to perform beyond yield during extreme storms. For example, SCRs that hang from a platform that experiences large heave motions during storms can be stressed beyond yield by displacement-controlled bending loads in the touchdown zone. The same riser will collect challenging fatigue load cycles. Fatigue loading can come from a number of sources including platform motions, vortex induced vibration and slugging. The production fluid from the reservoir can include CO₂, H₂S, and water along with the oil and gas.

As more operational experience is gained by the industry, it is becoming clear that maintaining control over corrosion through-out the life of subsea systems can be exceedingly difficult. When the internal chemistry of the produced fluid drifts out of control, pitting corrosion can result. When such pitting is identified by in-line inspection (ILI), the operator is faced with the task of evaluating the impact of the damage on the capacity of the riser to sustain fatigue loading and tolerate randomly occurring extreme loads in a chemical environment that may accelerate fatigue crack growth rates.

While basic fitness for service analysis is adequately covered in such codes as BS7910 [1] and API-RP-579 [2], the collection of issues unique to risers require special treatment. It is understood that fitness for service includes the evaluation of remaining strength capacity as well as the evaluation of the capacity to sustain fatigue loads. For risers and other fatigue loaded structure, the fatigue part of the assessment usually is more critical and is the primary focus of this paper. This paper

provides a methodology for making a fatigue life assessment of an SCR with corrosion pitting through the use of fatigue crack growth tools and provides an illustrative case study. The basis for the method is given as the process is delineated. While the basic process follows criteria outlined in ECA and FFS codes, key elements unique to the riser problem are addressed in details not available in the most commonly used codes. The methodology outlined assumes a steel riser.

Due to the potentially catastrophic consequence of any failure, and the high cost of construction, repair, and replacement, it is imperative that methods used to assess risers for fitness for service be completely reliable without being excessively conservative.

ABBREVIATIONS

ECA – Engineering Critical Assessment
EPFM – Elasto-Plastic Fracture Mechanics
FCG – Fatigue Crack Growth
FEA – Finite Element Analysis
FFS – Fitness for Service
FAD – Failure Assessment Diagram
IM – Integrity Management
KPI – Key Performance Indicator
SCF – Stress Concentration Factor
SCR – Steel Catenary Riser
TTR – Top Tensioned Riser
TDZ – Touch Down Zone
VIV – Vortex Induced Vibration

METHODOLOGY FOR ASSESSING FITNESS FOR SERVICE OF RISERS WITH CORROSION PITTING

When risers are designed it is common for corrosion to be accounted for by including a corrosion allowance in the wall thickness. However, when designing risers which are subject to fatigue loading from various sources, simply allowing extra thickness in the wall is inadequate to protect against the accelerated fatigue crack growth driven by corrosion. Uniform corrosion in risers is actually quite unusual. In most instances when corrosion occurs it is localized or in the form of pitting. While this type of wall loss may not present a risk from the standpoint of burst due to internal pressure, the geometry created by the corrosion creates a stress concentration that will speed the nucleation of cracks where fatigue loading is present. In the absence of an industry accepted method for assessing the fatigue and fracture performance of a structure containing corrosion pits, it is common practice to treat pits as planar flaws.

In the methodology outlined, corrosion pits are assessed as crack-like defects. The reasons for this approach are:

- It is conservative;
- The alternate approach for fatigue assessment, the SN approach, requires application of an appropriate SCF which requires knowledge of the radius at the base of the pit. This information is generally not available

- SN data in corrosive environments is more difficult to obtain than FCG data.
- Fatigue cracks can nucleate quickly from the bottom of corrosion pits.

The degree of conservatism inherent in this approach is quantified in the following section. The stress intensity factor of a very small crack at the bottom of a pit is compared to that of a crack with the same total depth.

COMPARISON OF CRACK WITH PIT

Treating pits and volumetric metal loss anomalies as though they are planar flaws is an acceptable assumption where the flaws are narrow and groove-like or where the root radius is sharp. Where the ILI tools cannot verify that the root radius is not sharp and cannot verify that cracks are not present, this conservative approach may be appropriate because of the criticality of the structure.

While assuming that the pit is a planar flaw is definitely a conservative assumption, it is important to understand quantitatively how conservative it is. To illustrate this, FEA is used to compare the stress intensity factor K_I of a series of small cracks at the bottom of a pit to the stress intensity of a corresponding series of full depth cracks. For the study, a 12.75 inch pipe with a 40 mm wall thickness is modeled. Loading is from internal pressure of 10 ksi. The loading is applied to the crack face and also to the inner face of the pipe and pit. The modeling is done in 2D with plane strain elements. In the first series a pit with radius of 5 mm is modeled with a range of crack lengths at the bottom. In the second configuration, a crack to the same total depth is modeled and loaded in the same way.

Figure 1 shows the model with the pit as constructed in ANSYS. The models are solved for the stress intensity factor (K_I) at the crack tip. The results are summarized in Table 1 and plotted in Figure 2. The comparison of the stress intensity factors calculated with these two configurations indicates that the assumption of treating pit as a full depth crack is an acceptable assumption where there is reason to believe that a small crack can quickly nucleate from the pit. By the time the crack has grown to 1.5 mm the stress intensity is much the same as a crack of the same depth.

It can be seen from Figure 2 that the stress intensity factor of 5 mm circular pit with a 0.5mm crack at the tip is only 77% of the stress intensity factor with a 5.5 mm deep crack. Clearly when the crack at the bottom of the pit is small, the conservatism of a full depth crack assumption increases.

The width of the pit also has an influence on the stress intensity at a small crack at the bottom of a pit. As the pit width increases, it becomes more conservative to treat pits as cracks. To illustrate this, the width of 5 mm deep pit is increased and solved for K_I . The results are plotted in Figure 3.

It is noted that in fatigue crack growth assessment, the rate of growth is roughly proportional to the stress intensity factor range to the power of 3. Small changes in the stress intensity

factor can have a significant impact on the predicted fatigue performance.

Configuration Type 1 5 mm pit + Crack		Configuration Type 2 Full depth crack	$\frac{K_{I-type1}}{K_{I-type2}}$
Crack depth (mm)	Total depth (mm)	(mm)	
0.5	5.5	5.5	0.77
1.0	6.0	6.0	0.90
1.5	6.5	6.5	0.96
2.0	7.0	7.0	0.98
2.5	7.5	7.5	0.99

Table 1 - Results from FEA of Corrosion Pit

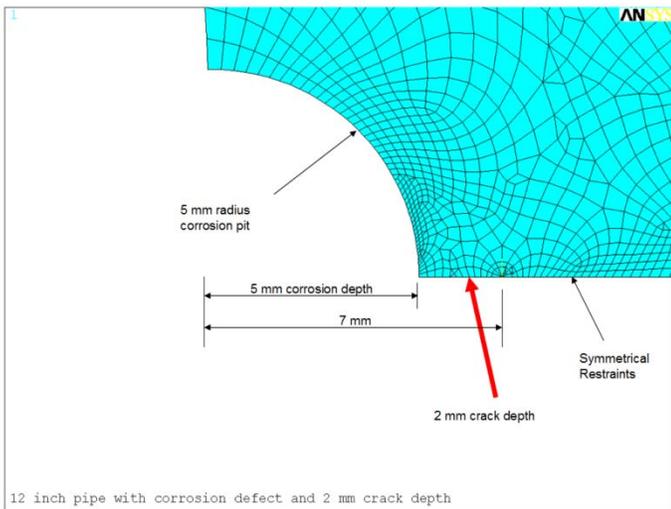
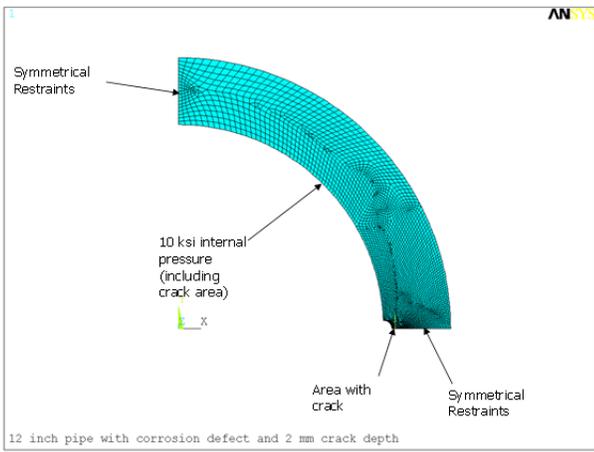


Figure 1- ANSYS model of pit with crack at tip

STRESS INTENSITY FACTOR (K_I) FOR CRACK AT BOTTOM OF PIT
5mm Circular Pit With Crack At Bottom In 12.75 Inch Pipe

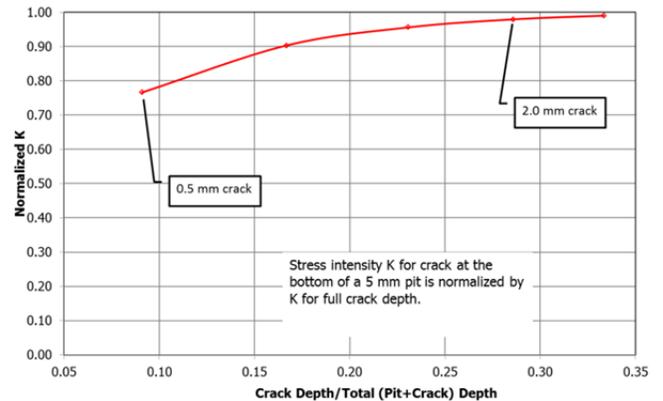


Figure 2 – Normalized K_I vs. Crack Depth / (Pit+Crack) Depth

EFFECT OF PIT WIDTH ON STRESS INTENSITY FACTOR
Corrosion Depth = 5 mm, Crack Depth = 0.5 mm, 1 mm, 2 mm

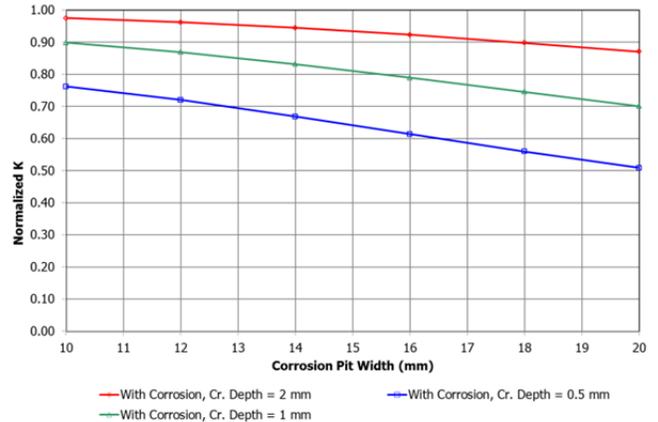


Figure 3 – Normalized K vs. Corrosion Pit Width

ISSUES COMPLICATING RISER FITNESS FOR SERVICE ASSESSMENT

This paper provides a methodology for making an assessment of an SCR with corrosion pitting through the use of an illustrative case study. Prior to developing the case study a series of key issues are discussed:

1 - Reference stress solutions

Both BS7910 and API579 use the reference stress approach to fracture analysis. At the time of this writing, there is a huge gulf between the two codes in the recommended formulations for reference stress for circumferential flaws in cylinders. The reference stress formulation in BS 7910 is derived from the work of Kastner [7]. It considers the ligament stress with the objective of predicting plastic collapse. Review of Kastner's paper shows a study pointed at a range of plastic collapse conditions covering burst in pipes with axial flaws, through wall flaws, as well as circumferential flaws. That study, however, has no experimental verification for circumferential surface flaws loaded in bending.

Other formulations for reference stress in pipes, notably API-RP-579 [2] and those in NASGRO [8] are based in global collapse of the section. They give solutions similar to a net section stress calculation and a much less conservative value of the limiting flaw size is predicted as a result.

A comparison of the reference stress from BS 7910 with the formulation from API-RP-579 is given in Figure 4. Note that a/t is the normalized crack depth in the BS 7910 equations. The plot clearly shows that with the BS 7910 equations the solutions diverge after a/t exceeds 0.5. The divergence is explained by examination of the BS 7910 solution which magnifies stress by dividing by the remaining wall thickness. In contrast, the API-RP-579 equation, only accounts for the stress increase in the global section caused by the loss of material. Further explanation is included in Appendix 2. In API-RP-579, the reference stress formulation comes from EPRI [6]. A revision to BS7910 currently under review includes the global reference stress solutions.

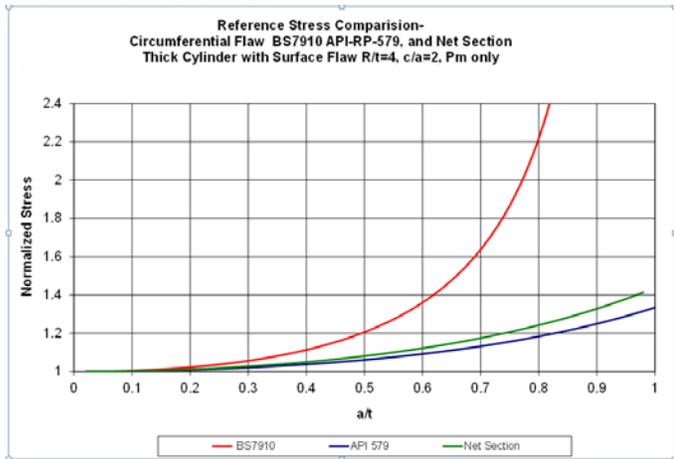


Figure 4 – Reference stress from API-RP579 vs. BS7910 for a thick walled pipe with circumferential crack.

In establishing which solutions are appropriate, some understanding of reference stress is required. In the simplest sense the reference stress is the stress in the pipe with the flaw accounted for. In that vein, it is sometimes described as the net section stress in the literature. However, even that definition is too imprecise to be of practical use. A more practical definition is given in equation (1). This definition allows FEA to be used to determine reference stress.

$$\sigma_{ref} = \frac{P}{P_0} \sigma_0 \quad (1)$$

Where:

- P = the applied load
- P_0 = the plastic collapse limit load for the cracked part assuming elastic perfectly plastic material
- σ_0 = yield stress

In ECA assessment the reference stress is used in more than one way, which may explain why there are different

approaches to determining it. In the problem at hand, reference stress is used:

- To predict local plastic failure of the remaining ligament ahead of the crack;
- To predict collapse of the full section;
- As the abscissa on the FAD diagram it is used to determine the allowable stress intensity ratio.

2 - Design stress beyond yield

Some SCRs are designed such that they undergo controlled bending and exceed yield during extreme storms using the principles of limit state design allowed in the governing codes [3]. Such designs are done cautiously to ensure there is control of the displacements and strain. In the basic ECA process, as well as in FFS studies, an initial flaw is grown through the application of cyclic load until the flaw is large enough that the section fails through fracture or by plastic collapse. This potential for failure is tested against the extreme load the structure is expected to encounter. Per both FFS codes [1][2] the 100 year extreme event is specified, but some in the offshore industry will use a 1000 year return period event. While the 1000 year event seems remote at first glance, there is a 2.5% chance that a structure with a life of 25 years will encounter it.

While limit state design is allowed by code, when the cross section of the pipe goes partially plastic, the evaluation of a defect in the section becomes more difficult and of course the limiting flaw becomes small. The basic approaches offered in BS7910 and API-RP-579 would predict failure when the reference stress exceeds yield stress or the flow stress. In the example developed here, only the final extreme load takes the section above yield. The cyclic loads which grow the flaw remain below yield.

When conducting FFS analysis for a section designed to yield, non-linear FEA analysis, which can incorporate the material stress-strain relations into the solution is required. Both global and local stress analysis should include the material non-linearity. This modeling will yield a stress profile through the section that is limited by local yielding. A profile of stress accounting for yielding is developed and then the plastic stress is applied to the cross section.

3 – Misalignment SCF

The handling of SCFs due to misalignment at the welds can be problematic in these cases. Unlike other stress concentrations, the misalignment SCF is not entirely secondary stress. To the extent it is primary stress it must be accounted for in the reference stress.

4. - FAD

EPFM, as employed in [1] and [2] is built around the failure assessment diagram. The FAD expresses fracture limit as a function of the load ratio L_r . Modern line pipe currently selected for use in SCRs is very ductile and sometimes even at the welds can show a minimum CTOD of 0.50mm.

Additionally, its stress strain curve is commonly characterized by a Lüders plateau. Because of this a Level 2B FAD, which uses the stress-strain curve to construct the FAD, or Level 3B FAD based on the use of the J-R curve is required. Constructing the FAD requires good knowledge of the stress strain curve up to about 2% strain.

Accounting for the Lüders plateau in the Level 2B FAD may introduce a penalty in the fracture limit beyond yield compared to the Level 2A FAD. This is illustrated in Figure 5.

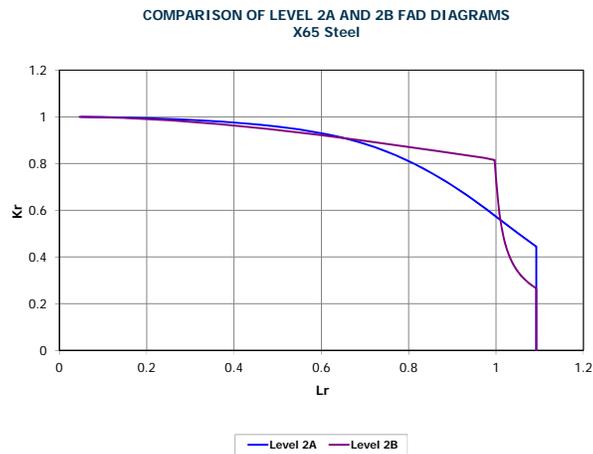


Figure 5 – Comparison of FAD for BS7910 FAD Level2A and Level2B.

5- Cyclic load

Many offshore structures, like pipelines and pressure vessels are essentially static or loaded with few significant load cycles. Risers, however, are dynamic structures. Fatigue loading is driven by first and second order vessel motions (seastate fatigue), VIV and slugging. Pressure cycling will contribute fatigue cycles in both the hoop and axial direction. During design, analysis is typically conducted to determine the long term loading stress histograms along the riser to predict fatigue using SN methods. In this paper typical histograms are used for illustration. While the SN approach will generally find the same locations are critical as when using the fatigue crack growth (FCG) approach, the introduction of pitting can change the critical locations.

6- Chemistry accelerated crack growth rates

Both CO₂ and H₂S can accelerate fatigue crack growth in carbon steel line pipe typically used in SCRs. In the case of H₂S, the effect will be more severe and the toughness may also be degraded. For both chemistries the effects vary with stress intensity, with minimal effect at low ΔK and very large effects at high ΔK. The effects are also very dependent upon pH and loading frequency, and testing of the specific steel at the specific chemical environment is recommended to avoid excessive conservatism. The fact that this effect changes with ΔK implies that it cannot be captured in the SN world with a knock-down factor. Adjustment to SN curves for corrosion fatigue should include a change in slope.

Crack Growth Curves and Test Data

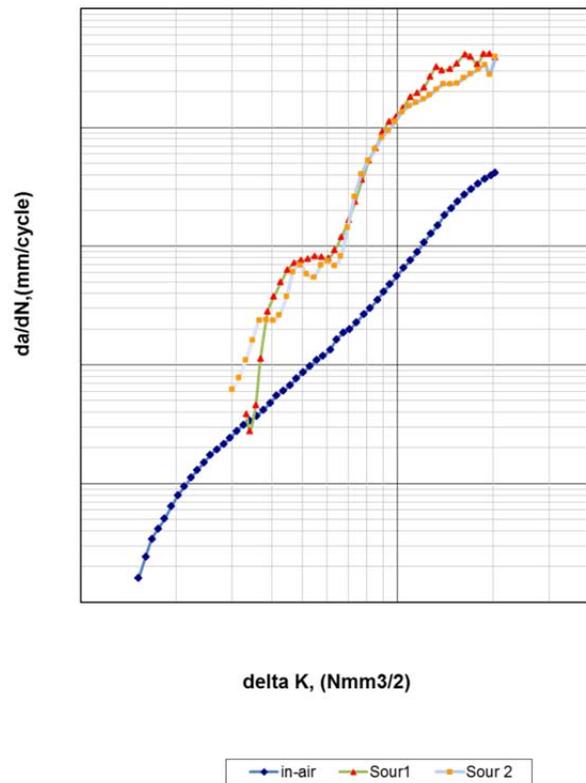


Figure 6 – Accelerated FCG in mild sour environment

7- High pressure and thick walled pipe

The offshore industry is currently developing reservoirs with wellhead pressures above 100 MPa. These systems require thick piping. Thick walled pipe, particularly where D/t is less than 12 puts the problem out of the bounds of the stress intensity (K) solutions given in BS7910 for cylinders with a circumferential flaw on the inner surface. Likewise for axial flaws on the ID of pipes the BS7910 solutions do not cover pipe with D/t less than 10. With higher pressures, it is not uncommon to find pipe with D/t less than 8. Alternative solutions are available such as those from API-RP-579 for the thicker pipe. Of course, those solutions are based on tabulated weight functions developed from FEA and are more difficult to implement in code than closed form solutions.

When pressures are high and pipe is thick walled, the influence of crack face pressure on the solutions can have significant impact on fracture, plastic collapse and fatigue performance. Figure 7 shows the effect of crack face pressure on the fracture limit of a riser pipe with an axial flaw. Figure 8 shows the effect of crack face pressure on the fracture limit of a riser pipe with a circumferential flaw. While the sensitivity of the circumferential flaw to the effects of the crack face pressure is less than for the axial flaw it is still significant.

Some codes such as BS 7910 do not include solutions that account for this effect. Others like API-RP-579 account for it in stress intensity, but not in reference stress, which can be unconservative. The application of crack face pressure to stress intensity factor and reference stress is given in Appendix 1 and 2.

The effect of crack face pressure on reference stress increases as the crack gets larger. For internal flaws it is inherently a function of a/t and a/c . In the absence of codified solutions, the FEA tool can be used to quantify the effect. Figure 15 gives the FEA results of reference stress component due to crack face pressure as a function of crack depth.

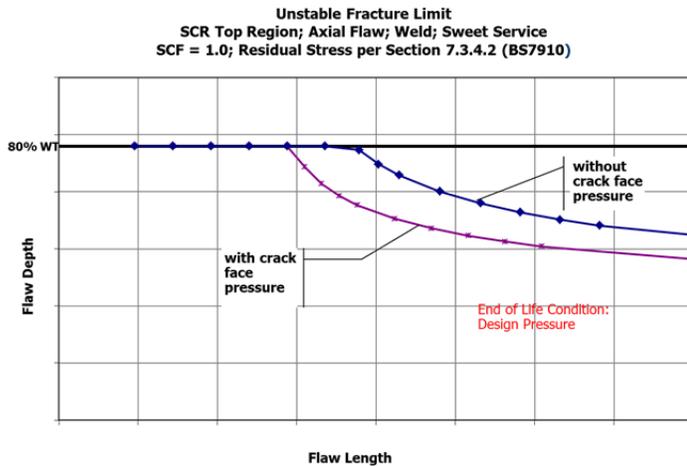


Figure 7 – Effect of crack face pressure with axial flaw [9]

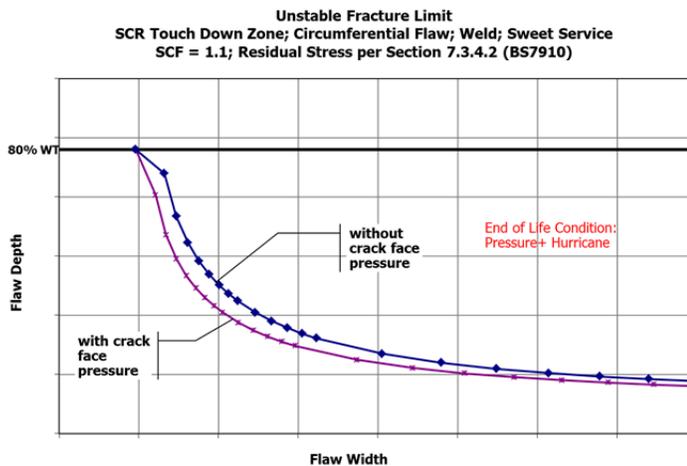


Figure 8 – Effect of crack face pressure for circumferential flaw [9]

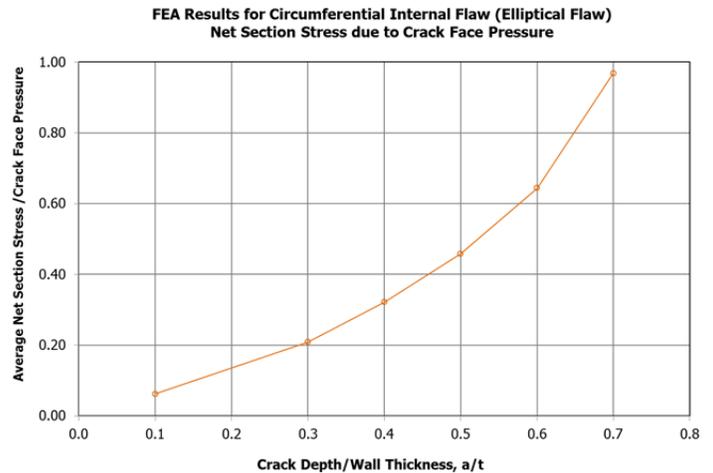


Figure 9 – Typical influence function derived from FEA to account for the effect of crack face pressure on reference stress.

8- Multi-stage life.

In evaluating fitness in a production riser, the analyst must account for changes in internal environment throughout the life of the field. This progression might include initial hydro-test events, normal production, in-place but shut down, sweet service, then increasing water cuts accompanied by mild souring. During the progression, there may be extreme events like hurricanes. There may be periods of the riser service where slugging occurs or where operational tactics like bull-heading impact the riser. In these successive environments the material properties such as CTOD, crack growth rates and yield strength will change with temperature and internal chemistry. The methodology used by the analyst should be able to simulate this order-dependent progression.

9- Inspection Data

Fitness for service analysis is typically conducted after inspection has shown some degradation. The degradation may be in the form of corrosion pitting or fatigue cracking. Unlike inspection during fabrication, inspection of an installed riser requires the use of a pig mounted tool. These fly-by inspections are inherently more challenging than inspections during fabrication. The measurement tolerance must be included in the estimate of the flaw size. If this tolerance is large, it can be a dominant influence on the outcome of the analysis. Although the current ILI tools can detect and size corrosion pitting damage, the reliability of the detection and sizing of fatigue cracking at girth welds or at corrosion pits has not yet been established.

EXAMPLE RISER SYSTEM DESCRIPTION

A production oil SCR supported by a semi-submersible is considered. The system is designed expecting initially sweet service, but mild souring in the second half of the production life is expected. Crack growth curves used are shown in Figure

10 with CO₂ inhibited sweet curve for long term sweet service. During hurricane events the BS7910 in-air (mean+2SD for R>=0.5) FCG curve [1] is adopted. For late life mild sour service the in-air da/dn rates are increased by a factor of 20 above at higher ΔK. The key material properties are given in Table 2. Level 2B ECA is conducted and bilinear stress-strain curve as shown in Figure 11 for X70 riser pipe is used. The objective is to determine what level of corrosion pitting the SCR is capable of tolerating and still be able to survive a 100 year hurricane.

Parameter	Value
Steel Grade	X70, de-rated to X65 in high temperature
Minimum yield strength, hot (MPa)	448
Minimum yield strength, cold (MPa)	482
Ultimate tensile strength, hot (MPa)	530
Ultimate tensile strength, cold (MPa)	565
Elastic modulus at room temp (MPa)	207,000
Poisson's ratio	0.3

Table 2– Physical Data for SCR [4]

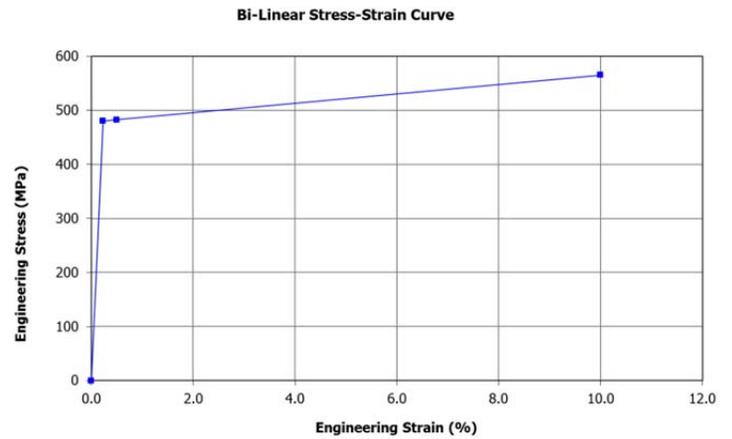


Figure 11 – Bilinear Stress-Strain Curve for X70 Riser Pipe

METHODOLOGY

In this riser system example calculation, flaws in both the circumferential and axial orientations are considered. The corrosion pits are modeled as planar flaws. It is recognized that this approach is conservative and depending upon the pit morphology may be excessively conservative. Nevertheless, in the absence of an industry accepted method of analyzing risers containing corrosion pits in fatigue service, this approach is selected. The riser TDZ and top are identified as two of the most critical hot-spots for crack growth. Full crack face pressure is applied conservatively in both the stress intensity factor and the reference stress calculations. API-RP-579 reference stress is adopted. Misalignment SCF is taken into account in both the stress intensity factor and the reference stress calculations. When the loading pushes stresses past yield locally (in 100 year hurricane), a Neuber correction [5] is applied to incorporate the effect of misalignment. Load sequence has a significant effect in crack growth and evaluation of the crack growth cycle by cycle is conducted in this example calculation.

Loads on the riser system are characterized by two types:

- Extreme loads: loads that define the end of life condition for a flaw. These are based on an extreme event, which if it were to impact a flaw of certain critical size, would result in an unstable growth of the flaw to failure;
- Long term loads: loads that occur on a regular basis and are a result of everyday occurrence. These are typically cyclical loads that result in fatigue crack propagation.

Loading conditions for the riser system considered are summarized in Table 3.

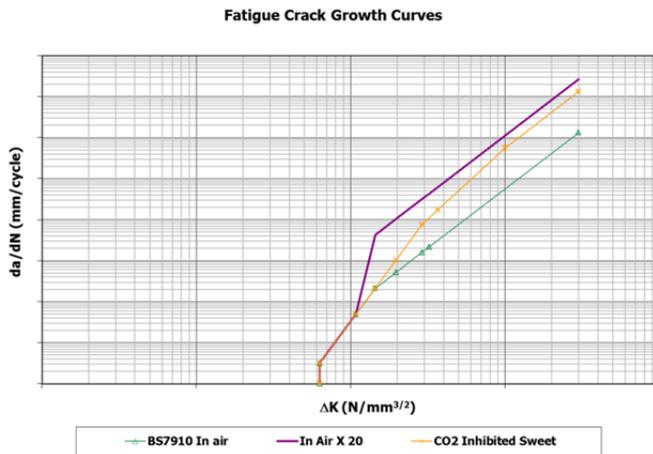


Figure 10 – Sweet and Sour Service Crack Growth Curves

Flaw Orientation	Extreme Loads		Long Term Loads
	TDP	Top	TDP/Top
Circumferential	100yr Hurricane	10yr Winter Storm	Fatigue Seastate Loading, VIV
	Ambient Pressure	Design Pressure	Operational Internal Pressure
Axial	Design Pressure	Design Pressure	Operational Internal Pressure

Table 3 – Extreme and Long Term Loads

Rainflow counting method is used to generate the long-term load histograms from first and second order vessel motion loading.

RESULTS

Fracture analysis is conducted to determine the limiting flaw sizes that cause unstable fracture for a range of aspect ratios. Additionally the tolerable initial flaw sizes that result in 6 years, 8.5 years, 15 years, and 27 years of target fatigue lives due to long term loading, with a factor of safety of 5, are determined. The limiting flaw sizes are shown in Figure 12 for circumferentially oriented flaws, and are shown in Figure 13 for axially oriented flaws. The allowable initial flaw plots define the maximum starting flaw sizes that will grow to critical sizes in defined periods (target lives).

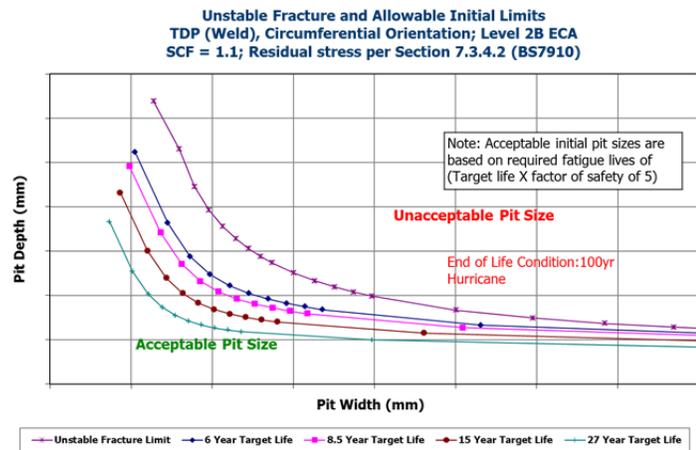


Figure 12 – Limiting Unstable Fracture and Allowable Initial Circumferential Flaws in Weld Metal at TDP

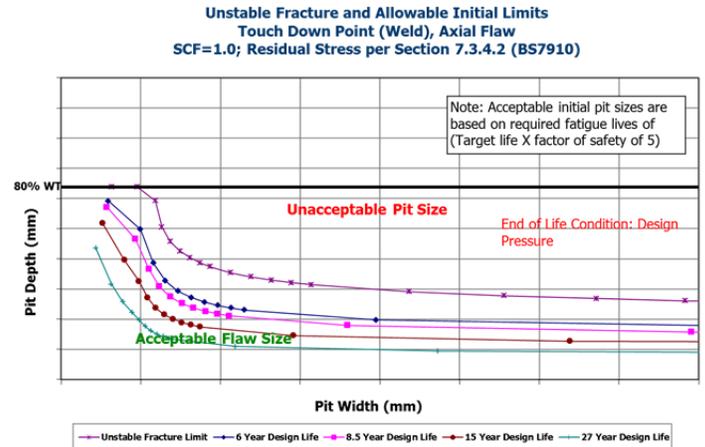


Figure 13 – Limiting Unstable Fracture and Allowable Initial Axial Flaws in Weld Metal at TDP

The tolerable initial flaw sizes that result in following target fatigue lives with a factor of safety of 5 are determined and shown in Figure 14 when the occurrence of 100yr hurricane is taken into account:

- 3 hours of 100yr hurricane;
- 5 years of long term loading followed by 3 hours of 100yr hurricane;
- 10years of long term loading followed by 3 hours of 100yr hurricane;
- 6 hours of 100yr hurricane;
- 10years of long term loading followed by 6 hours of 100yr hurricane.

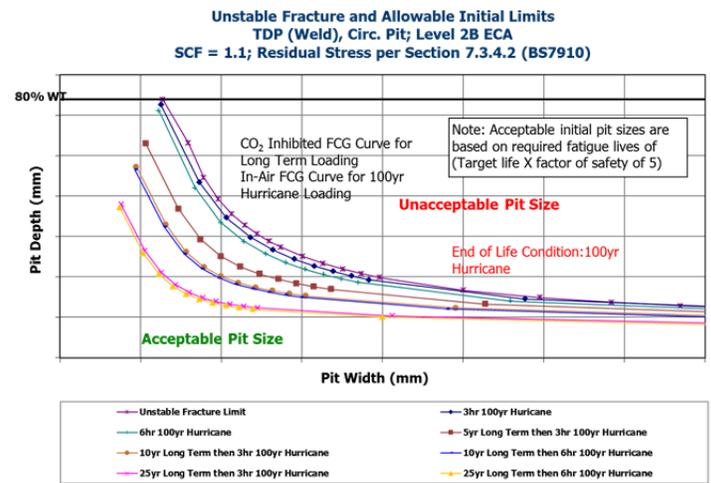


Figure 14 – Limiting Unstable Fracture Sizes and Tolerable Initial Circumferential Pits along Various Target Lives due to Long Term Loading Followed by 100yr Hurricane in Weld at TDZ

DISCUSSION

To determine if and when unstable fracture occurs, inspection data (pit depth/width) is superimposed on the

limiting unstable fracture and allowable initial flaw size plots. If the inspection data fall below the limiting unstable fracture curve, unstable fracture is not expected to occur at the time of inspection. If the inspection data fall below the allowable initial curve of a certain target life, unstable fracture is not expected to occur after the specified target life.

As the FEA results presented in the paper illustrate, small cracks located at pits generally have lower stress intensity than a corresponding full depth crack. To reduce the conservatism inherent in treating corrosion pits as planar flaws, it is possible to develop and apply factors to account for the influence of pit geometry on the K solution. A matrix of influence factors can be obtained by FE analysis of different pit width, pit depth, and crack depth combination. Ongoing work in this area will be presented in a future paper.

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APPENDIX 1– STRESS INTENSITY ADJUSTMENT TO ACCOUNT FOR CRACK FACE PRESSURE

The effect of crack face pressure is derived for a through-wall flaw in a plate using the superposition theory. Diagrams A and B in Figure 15 illustrate that the stress field in a loaded uncracked plate will be equivalent to that in a loaded cracked plate where σ is applied to close the crack. Also consider that the stress intensity at the crack tip in B is zero. This means that the stress intensity at the crack tip for the sum of D and E must equal zero. Therefore D and E are equal and opposite. Then note that the negative of D is in fact the crack face pressure effect. One can conclude that the crack face pressure case will give the same stress intensity as the uniform stress at a distance as shown in diagram D.

This means that the effect of crack face pressure is equivalent to a uniform stress of the same magnitude as the pressure applied at a distance. FEA shows that this holds for surface flaws as well.

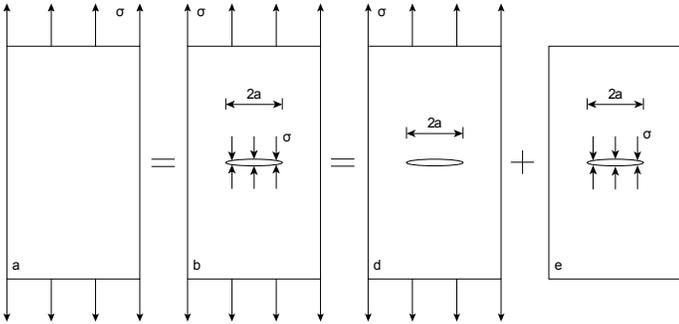


Figure 15 – Superposition of stress to account for crack face pressure

APPENDIX 2– REFERENCE STRESS ADJUSTMENT TO ACCOUNT FOR CRACK FACE PRESSURE

The reference stress equations in BS7910 are based on the work of Kastner and posit local collapse which results in conservative solutions for thicker pipe. Here the analyst can adopt the reference solutions from API-RP-579. A comparison is shown in Figure 4 which shows how much the solutions diverge for deeper flaws. Effectively the API solution is close to the net section solution as is the reference solution from NASGRO [8]. The relevant equations for circumferential flaws are given below:

BS7910 Reference stress for surface flaws in plates:

$$\sigma_{ref} = \frac{P_b + \{P_b^2 + 9P_m^2(1 - \alpha'')^2\}^{0.5}}{3(1 - \alpha'')^2} \quad (2)$$

Where:

Pm = primary membrane stress

Pb = primary bending stress

$$\alpha'' = \frac{\frac{a}{B}}{1 + \frac{B}{c}} \quad \text{for } W \geq 2(c + B)W \quad (3)$$

$$\alpha'' = \frac{2ac}{BW} \quad \text{for long flaws } W < 2(c + B) \quad (4)$$

BS7910 Reference stress for surface flaws in cylinders:

$$\sigma_{ref} = \frac{P_m \left[\pi \left(1 - \frac{a}{B} \right) + 2 \left(\frac{a}{B} \right) \sin \left(\frac{c}{r} \right) \right]}{\left(1 - \frac{a}{B} \right) \left[\pi - \left(\frac{c}{r} \right) \left(\frac{a}{B} \right) \right]} + \frac{2P_b}{3(1 - \alpha'')^2} \quad (5)$$

API-RP-579 Reference stress for cylinders:

$$\sigma_{ref} = \frac{P_b + \{P_b^2 + 9Z^2 P_m^2 (1 - \alpha'')^2\}^{0.5}}{3(1 - \alpha'')^2} \quad (6)$$